Problem 12-12

Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h^2 along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time?

Solution

The relationship between acceleration and velocity is

$$a = \frac{dv}{dt} = 6000.$$

Integrate both sides with respect to t to get $v(t)$.

$$v(t) = 6000t + C_1$$

Use the fact that the initial speed is 70 km/h to determine C_1.

$$v(0) = C_1 = 70$$

The car's velocity (in kilometers per hour) is then

$$v(t) = 6000t + 70.$$

Set $v(t) = 120$ and solve the equation for t.

$$120 = 6000t + 70$$

$$50 = 6000t$$

$$t = \frac{1}{120} \text{ h}$$

Therefore, it will take half a minute, or 30 seconds, for the car to reach 120 km/h. The relationship between velocity and position is

$$v = \frac{ds}{dt} = 6000t + 70.$$

Integrate both sides with respect to t.

$$s(t) = 3000t^2 + 70t + C_2$$

Assume the car is at $s = 0$ when $t = 0$ to determine C_2.

$$s(0) = C_2 = 0$$

The car's position (in kilometers) is then

$$s(t) = 3000t^2 + 70t.$$

Plug in $t = 1/120$ to find out how far the car travels during this time.

$$s\left(\frac{1}{120}\right) = 3000\left(\frac{1}{120}\right)^2 + 70\left(\frac{1}{120}\right) = \frac{19}{24} \text{ km} \approx 0.792 \text{ km}$$